

A Fast Index Assignment Method for Robust Vector Quantisation of Image Data

N. L. C. Talbot

G. C. Cawley

Institute of Food Research
Norwich Research Park
Norwich
NR4 7UA, UK.

School of Information Systems
University of East Anglia
Norwich
NR4 7TJ, UK.

Abstract

Vector quantisation is a widely used technique in low-bit rate coding of speech and image data, but is highly sensitive to noise in the transmission channel. If the reference vector recalled by a corrupted index differs greatly from the intended reference vector, image quality can be degraded quite dramatically. The index assignment (IA) process attempts to re-order the code book so as to minimise the effects of errors introduced in the transmission channel, by assigning indices with similar binary patterns to similar reference vectors, usually at considerable computational expense. This paper describes a fast, novel index assignment algorithm based on Hall's solution to the quadratic assignment problem [6].

vector \mathbf{w}_i . The incoming vector can then be represented by the binary index i , resulting in a considerable reduction in the required bit rate. Vector quantisation is however very sensitive to errors in the transmitted codeword, due to noise in the transmission channel, as the reference vectors retrieved by the intended and corrupted indices might be very different. The index assignment (IA) process attempts to re-order the code book, such that similar reference vectors are recalled by indices with similar binary patterns [4], minimising the effects of an error in the transmitted index.

This paper presents the results obtained using a novel index assignment method [3, 9], based on Hall's solution to the quadratic assignment problem [6], for robust gain-shape vector quantisation of image data.

1: Introduction

Given p -dimensional unlabelled data $Y = \{\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_m\} \subset \mathbb{R}^p$ representative of a data manifold $V \subseteq \mathbb{R}^p$, the process of vector quantisation [5] attempts to partition V into a number of sub-regions V_i using a finite set of reference or *code book* vectors, $W = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \dots, \mathbf{w}_n\} \subset \mathbb{R}^p$. The values of the reference vectors are chosen so as to minimise the quantisation error, measured according to a distance metric, between a training vector \mathbf{y}_i and the best matching reference vector $\mathbf{w}_{(y_i)}$ over all vectors in Y . An incoming vector $\mathbf{v} \in V_i$ is mapped onto the most similar reference

2: Quadratic index assignment

The aim of index assignment can be viewed as placing the $n = 2^r$ reference vectors in an r dimensional binary space, representing the transmitted code words or indices, such that similar reference vectors are placed in similar locations in this r dimensional binary space. In the approach described here, Hall's solution to the quadratic assignment problem [6] is used to obtain a suitable placement for each of the reference vectors in an r dimensional *continuous* space, which is then quantised using a simple recursive partitioning algorithm to produce a near optimal reordering of the code book.

2.1: Quadratic assignment

Many practical problems, for instance VLSI cell placement [8], can be considered as special cases of the quadratic assignment problem. The quadratic assignment problem can be simply stated as follows: If x_{ik} represents the x_k co-ordinate of node i , and c_{ij} represents the ‘connection strength’ between nodes i and j , find the optimal placement for each node so as to minimise

$$\frac{1}{2} \sum_i \sum_j \sum_k (x_{ik} - x_{jk})^2 c_{ij}$$

Hall [6] presents a solution to this problem by constructing the ‘disconnection’ matrix, \mathbf{B} , where

$$b_{ij} = \begin{cases} \sum_k c_{ik} & i = j \\ -c_{ij} & i \neq j \end{cases}$$

The matrix \mathbf{B} is semi-positive definite with eigenvalues $0 = \lambda_1 < \lambda_2 \leq \lambda_3 \leq \dots \leq \lambda_n$. The solution vectors, \mathbf{x}_k , are then the eigenvectors of \mathbf{B} corresponding to the r smallest non-zero eigenvalues.

2.2: Index assignment

The following procedure is used to reorder the code book using Hall’s quadratic assignment algorithm: given $n = 2^r$ reference vectors, construct the disconnection matrix such that the connection strength c_{ij} is large if \mathbf{w}_i and \mathbf{w}_j are similar, and small if they are dis-similar. For initial experiments, the connection strength is given by:

$$c_{ij} = \begin{cases} \frac{1}{\|\mathbf{w}_i - \mathbf{w}_j\|} & i \neq j \\ 0 & i = j \end{cases}$$

A recursive partitioning algorithm is then used to convert the continuous valued solution to a set of binary indices used to re-order the code book, as follows:

1. Construct the $(n \times (r + p))$ augmented matrix

$$\mathbf{Z} = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1r} & \mathbf{w}_1^T \\ x_{21} & x_{22} & \dots & x_{2r} & \mathbf{w}_2^T \\ \vdots & & \ddots & \vdots & \vdots \\ x_{n1} & x_{n2} & \dots & x_{nr} & \mathbf{w}_n^T \end{pmatrix}$$

2. For each $i = 1, \dots, r$
set $N = 2^{r-i+1}$
3. For each $j = 1, \dots, 2^{i-1}$

sort rows $(j - 1) \times N + 1$ to $j \times N$ of \mathbf{Z}
so that column i is in piecewise ascending order

4. Remove the first r columns of the matrix \mathbf{Z}

\mathbf{Z} now represents the sorted code book.

3: Method

For evaluation, a codebook was created from images taken from the ORL face database [1]. Each image was first reduced to 90×111 pixels, by deleting the first and last columns and the last row, and then divided into 3×3 pixel blocks. A gain-shape vector quantisation scheme was implemented, and so the resulting training vectors were normalised to have unit length before generating a code book of 256 reference vectors using the Linde-Buzo-Gray (LBG) algorithm [7]. The codebook was then re-ordered using the proposed method based on Hall’s solution to the quadratic assignment problem and also, for comparison, using a simple pair-wise interchange algorithm with simulated annealing [2].

4: Results

Figure 1(a) shows an image taken from the Olivetti Research Laboratory (ORL) face database, encoded using a gain-shape vector quantisation scheme with an unsorted codebook of 256 reference vectors. Figure 1(b) shows the same image reconstructed from indices corrupted by random noise, where a single bit error is introduced randomly into one in ten of the code book indices, at a random bit position. This figure clearly illustrates the sensitivity of vector quantisation schemes to noise in the transmission channel, as single bit errors in the transmitted indices are seen to result in very noticeable distortions in the reconstructed image. Figures 1(c) and (d) show images quantised using code books ordered using the proposed quadratic index assignment method and a pair-wise interchange algorithm with simulated annealing respectively, with the same pattern of single bit errors in the sequence of code book indices. It can easily be seen that both index assignment procedures produce similar improvements in image quality; however the quadratic index assignment method operates around two orders of magnitude faster than the more conventional method based on simulated annealing.

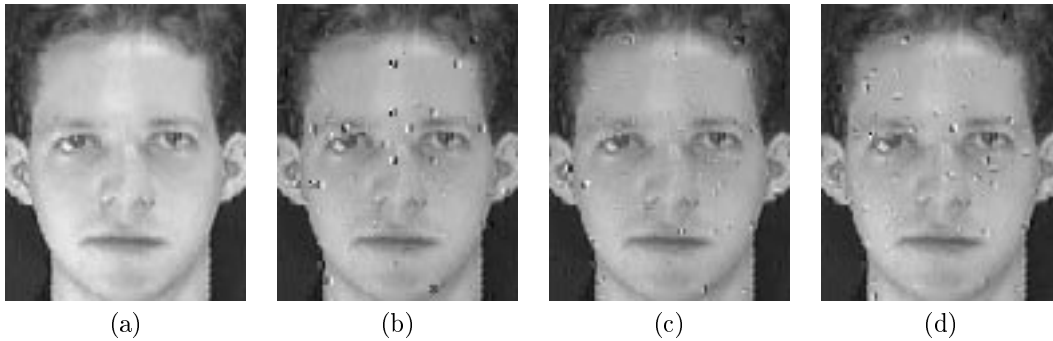


Figure 1: (a) vector quantised image, vectors (b), (c) and (d) vector quantised images with a fixed pattern of single bit errors in the sequence of indices, using (b) an unsorted code book, and code books ordered using the proposed algorithm (c) and random pair-wise interchange with simulated annealing (d).

Figure 2 shows a graph of the average root mean square error in the reference vectors due to errors introduced in the transmission channel. For an unsorted codebook (labelled ‘VQ’), the error is almost constant regardless of the number of errors in the transmitted index, so that on average, a single bit error would cause the same distortion in the reconstructed image as if all eight bits of the index had been corrupted. For ordered codebooks (labelled ‘QIA’ for the proposed quadratic index assignment algorithm and ‘SAIA’ for the method based on simulated annealing), the distortion in the reconstructed image is much lower for single bit errors than for larger Hamming distances. This greatly improves image quality as single bit errors in the transmitted index are likely to be by far the most common.

5: Conclusions

A novel index assignment algorithm is presented, based on Hall’s solution to the quadratic assignment problem. The resulting code book ordering is of comparable quality to that generated using a conventional simulated annealing algorithm, but is obtained at a greatly reduced computational expense.

References

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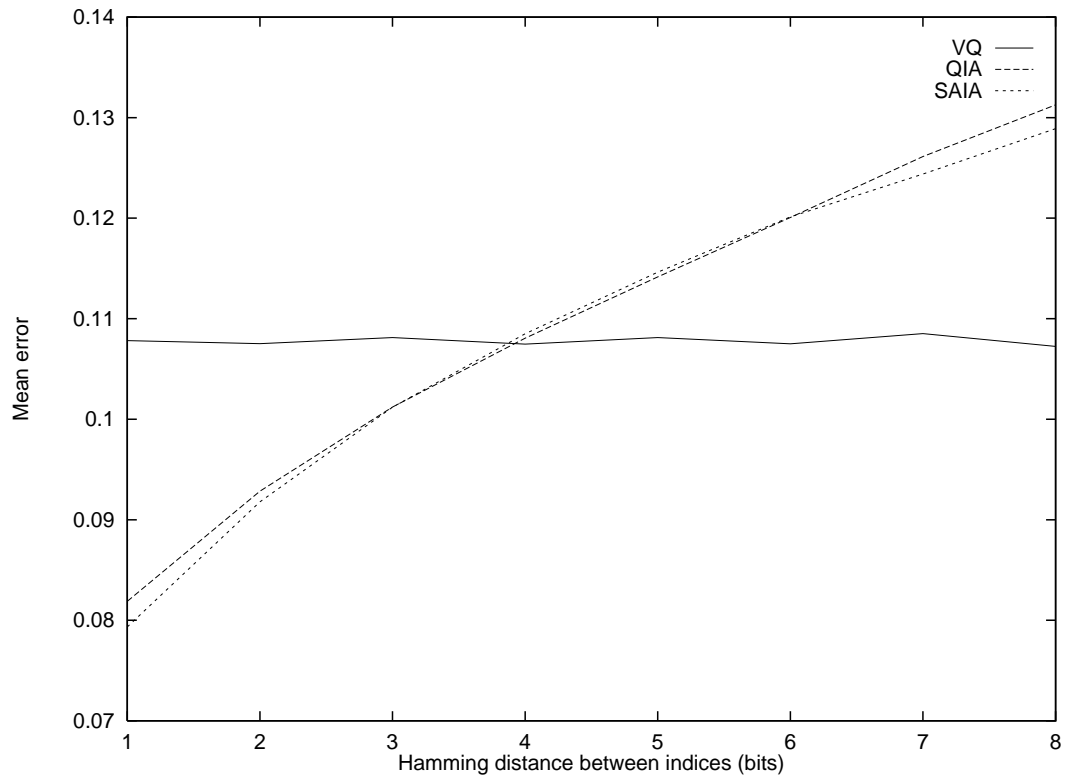


Figure 2: A graph of the average root mean square error in the reference vectors due to errors introduced in the transmission channel, for an unsorted codebook (VQ) and codebooks ordered using the proposed algorithm (QIA) and a simulated annealing algorithm (SAIA). The error is almost constant for the unsorted codebook, whereas the sorted codebooks have a much lower distortion for single bit errors than for larger Hamming distances