A Fast Index Assignment Algorithm for Vector Quantization over Noisy Transmission Channels

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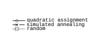
List of Captions

Figure 1 : Graph of mean error power against Hamming distance, for an un-ordered codebook, and codebooks ordered using simulated annealing and quadratic assignment algorithms.



Abstract

Vector quantisation, a widely used technique in low-bit rate coding of speech signals, is highly sensitive to errors in the transmitted codeword caused by noise in the transmission channel. This paper describes an efficient index assignment algorithm, based on Hall's solution to the quadratic assignment problem, used to re-order the codebook such that the effect of transmission errors is minimised.

Introduction

Given p dimensional unlabelled data $Y = \{\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_m\} \subset I\!R^p$ representative of a data manifold $V \subseteq I\!R^p$, the process of vector quantisation [1] attempts to partition V into a number of sub-regions V_i using a finite set of reference or *code book* vectors, $W = \{\boldsymbol{w}_1, \boldsymbol{w}_2, \boldsymbol{w}_3, \ldots, \boldsymbol{w}_n\}$. The values of the reference vectors are chosen so as to minimise the quantisation error, measured according to a distance metric, between a training vector $\boldsymbol{y}_i$ and the best matching reference vector $\boldsymbol{w}_{(y_i)}$ over all vectors in Y. An input vector $\boldsymbol{v} \in V_i$ is mapped onto the most similar reference vector $\boldsymbol{w}_i$. The incoming vector can then be represented by the index i.

Vector quantisation is however very sensitive to errors in the transmitted codeword, caused by noise in the transmission channel, as the reference vectors recalled by the intended and corrupted indices might be very different. The index assignment (IA) process attempts to re-order the codebook, such that similar reference vectors are recalled by indices with similar binary patterns [2]. This minimises the effect of an error in the transmitted codeword, as the reference vector corresponding to the corrupted index is made as similar as possible to the reference vector represented by the intended index.

Model

The index assignment process, where a code book is re-ordered such that similar reference vectors are represented by indices with similar binary patterns, is analogous to the problem of placement in VLSI circuits, where modules are placed on a two-dimensional surface so as to minimise the total wire length [3]. The aim of Index Assignment can be viewed as placing the $n = 2^r$ reference vectors in the r-dimensional binary space described by the codewords, such that similar reference vectors are located in similar locations. Like VLSI cell placement, the solution can be found using Hall's [4] solution to the quadratic assignment (QA) problem.

The aim of quadratic assignment is to place n nodes in an r dimensional space such that

$$z = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{r}(x_{ki} - x_{kj})^2 c_{ij}$$

is minimised, where x_{ki} represents the kth co-ordinate of node i, and c_{ij} is the *connection strength* between nodes i and j. To avoid the trivial solution $\boldsymbol{x}_k = \mathbf{0}$, the constraints $\boldsymbol{x}_k^T\boldsymbol{x}_k = 1$ are imposed. Hall [4] defines the 'disconnection' matrix, $\boldsymbol{B}$, to be:

$$b_{ij} = \begin{cases} -c_{ij} & i \neq j \\ \sum_{l=1}^{n} c_{il} & i = j \end{cases}$$

and shows that the matrix $\boldsymbol{B}$ has eigenvalues $0 = \lambda_0 < \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_{n-1}$. The solution vectors, $\boldsymbol{x}_k$ are given by the normalised eigenvectors corresponding to the eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_r$. The eigenvalues and eigenvectors of the symmetric disconnection matrix can be determined using Householder's algorithm ($O(4n^3/3)$) and the QL algorithm with implicit shifts ($O(3n^3)$) [5]. For the index assignment problem, so that similar reference vectors exhibit a high connection strength, c_{ij} is simply the reciprocal of the Euclidean distance between reference vectors i and j.

$$c_{ij} = \frac{1}{\|\boldsymbol{w}_i - \boldsymbol{w}_j\|}$$

The resulting continuous valued vectors $\boldsymbol{x}_i$ are then transformed into the appropriate binary codewords by a simple recursive partitioning procedure.

Results

For evaluation, a codebook of 256 reference vectors, compiled from the data obtained from line spectral pair (LSP) analysis of speech from the DARPA/TIMIT speech corpus was re-ordered using the quadratic assignment method and a conventional pair-wise exchange algorithm with simulated annealing [6]. The codebooks generated using the quadratic assignment and simulated annealing algorithms, are similarly well-ordered, as illustrated by Figure 1. The graph shown in Figure 1 was created by simulating, for each reference vector, 1, 2, ..., 8 bit-errors in every possible bit position and noting the resulting spectral distortion. The randomly ordered codebook shows practically no change in performance as the number of errors increases, as expected, whereas the two ordered codebooks show significantly reduced spectral distortion for single bit errors. However, the quadratic assignment method was found to be around two orders of magnitude faster than the conventional approach based on simulated annealing.

Conclusions

A novel index assignment algorithm is presented, based on Hall's solution to the quadratic assignment problem. The resulting codebook structure is comparable to that generated using a conventional simulated annealing algorithm, but is obtained at a greatly reduced computational expense.

References

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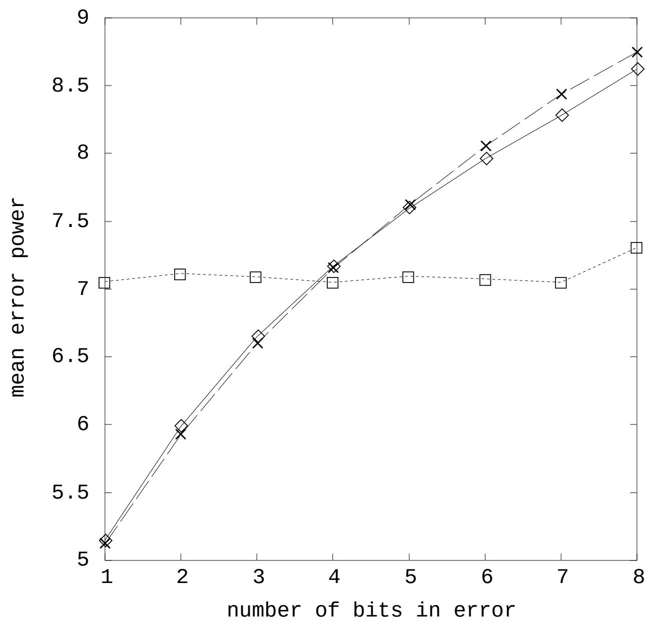

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quadratic assignment
simulated annealing
random

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